Birzeit University
Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Communication Systems ENEE 339
Final Exam

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Problem 1: 20 Points

Let $m(t)$ be a baseband signal defined as

$$m(t) = \begin{cases} 0.5 & -1 \le t \le 1 \\ 0 & otherwise \end{cases},$$

a. Find the energy in $m(t)$.
b. Find the Null-to-Null Bandwidth of m(t).
c. If $m(t)$ is applied to an FM modulator with a sensitivity constant $k_f = 10\ Hz/V$ along with the carrier $4\cos(2\pi \times 20 \times t)$ to produce the modulated signal $s(t)$. Find and sketch the instantaneous frequency of $s(t)$ for $-3 \le t \le 3$.

Problem 2: 20 Points

The communication system shown in figure 1 below is designed such that the passband bandwidth of the Low Pass filter is 4KHz and the sampling frequency (fs) of 8KHz. The signal $m(t) = 2\cos(2\pi \times 3000 \times t) + 4\cos(2\pi \times 6000 \times t)$ is applied to the input of the system. Answer the following questions assuming ideal filters are used.

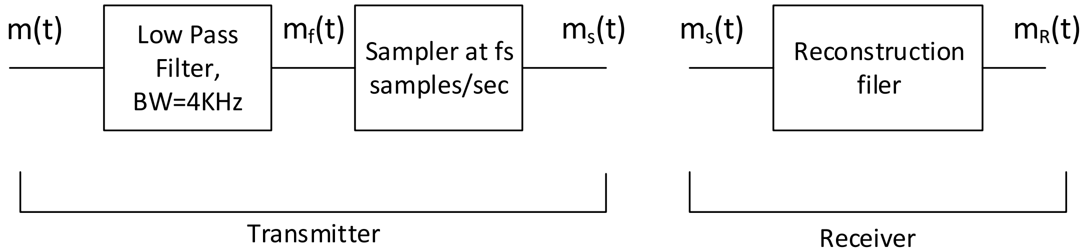


Figure 1: Block diagram of communication system for Problem 2

a. Find the signal filtered signal $m_f(t)$.
b. Plot the amplitude spectrum of the filtered signal $m_f(t)$.
c. Plot the amplitude spectrum of the sampled signal $m_s(t)$.
d. What is the type of the Reconstruction filter at the receiver?
e. What is the passband bandwidth of the Reconstruction filter at the receiver?
f. Write down the reconstructed signal $m_R(t)$ at the output of the receiver in the time domain.

Problem 3: 20 Points

The waveform m(t) shown in Figure 2.a is a segment of a voice speech signal that has a maximum frequency component at 4KHz. The dynamic range of the Uniform quantizer is from -2 to 2 volts. Assume the waveform is processed as described in the block diagram shown in figure 2.b, answer the following:

a. Determine the sampled signal values; $m_s(nT)$ for $0 \leq t \leq 0.5$.
b. Determine the quantized signal values; $m_q(nT)$ for $0 \leq t \leq 0.5$.
c. Determine the binary signal values at the output of the 2 bit encoder for the quantized samples of part b(m_b),
d. Determine the output bit rate at the quatizer output in bits per second; (R_b),
e. If the binary data of Part c are transmitted using binary phase shift keying, find the required transmission bandwidth.

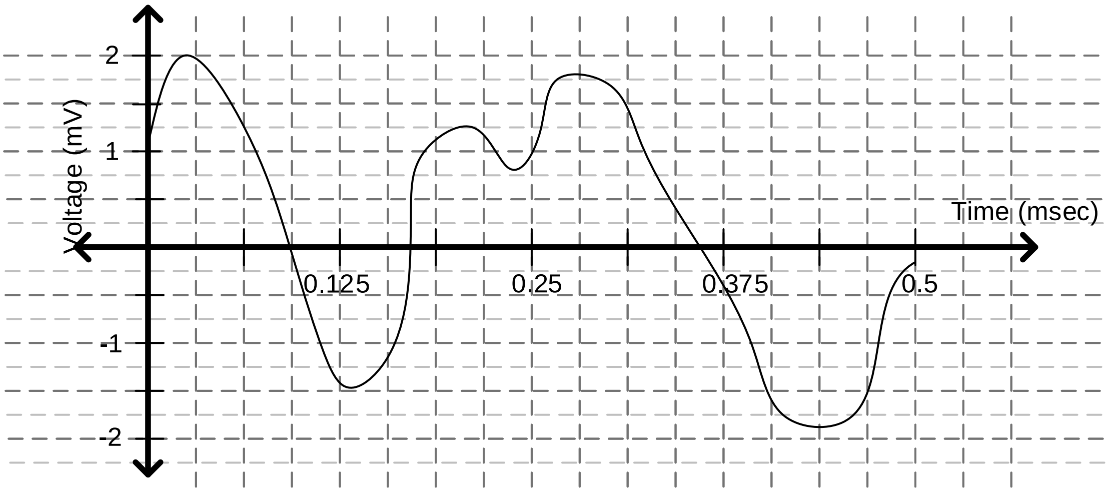


Figure 2.a: m(t) waveform

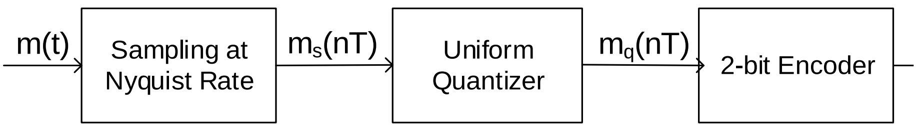


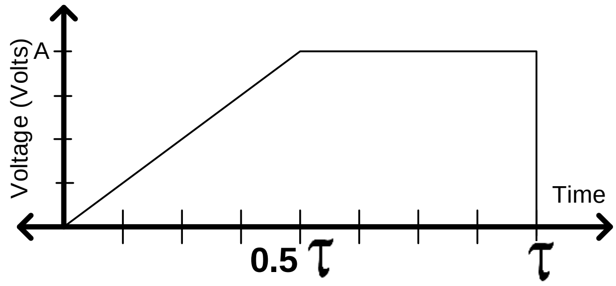
Figure 2.b: PCM system

Problem 4: 20 Points

The binary digital communication signaling scheme, discussed in class, employs the following two equally probable signals $s_1(t)$ and $s_2(t) = -s_1(t)$ to represent binary logic 1 and 0 respectively over a channel corrupted by AWGN with power spectral density $N_0/2$ W/Hz. Here,

$$s_1(t) = \begin{cases} A\dfrac{2t}{\tau}, & 0 \le t \le \tau/2 \\ A, & \tau/2 \le t \le \tau \end{cases}$$

Where τ is the binary symbol duration.



a. Find and sketch the impulse response, $h(t)$, of the matched filter, designed to minimize the system probability of error.
b. Find the optimum threshold used at the receiver
c. Sketch the optimum receiver highlighting its basic components

Problem 5: 20 Points

The binary orthogonal frequency shift keying (FSK) signaling scheme, discussed in class, employs the following two equally probable signals $s_1(t)$ and $s_2(t)$ to represent binary logic 1 and 0 respectively over a channel corrupted by AWGN with power spectral density $N_0/2$ W/Hz:

$$s_1(t) = 4\cos(2\pi f_1 t), \qquad 0 \le t \le \tau$$
$$s_2(t) = 4\cos(2\pi f_2 t), 0 \le t \le \tau$$

where $f_1 = nR_b$, $f_2 = mR_b$, $R_b = 1/\tau$, and n and m are integers.

a. Find the system average probability of error
b. If the bit error probability is not to exceed 8.8417x 10^{-5}, find the maximum allowable data rate $R_b = 1/\tau$ if $N_0 = 0.001$ (use the attached Q-function table)
c. Find the 90% bandwidth when $f_1 = 8KHz$, $f_2 = 2KHz$, and $R_b = 1Kbps$

Good Luck

TABLE A6.4 *Trigonometric Identities*

$\exp(\pm j\theta) = \cos\theta \pm j\sin\theta$

$\cos\theta = \frac{1}{2}[\exp(j\theta) + \exp(-j\theta)]$

$\sin\theta = \frac{1}{2j}[\exp(j\theta) - \exp(-j\theta)]$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta - \sin^2\theta = \cos(2\theta)$

$\cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)]$

$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)]$

$2\sin\theta\cos\theta = \sin(2\theta)$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$

$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$

$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$

TABLE A6.2 *Fourier-Transform Pairs*

Time Function	*Fourier Transform*
$\mathrm{rect}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}\,(fT)$
$\mathrm{sinc}\,(2Wt)$	$\frac{1}{2W}\mathrm{rect}\left(\frac{f}{2W}\right)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a + j2\pi f}$
$\exp(-a\lvert t\rvert), \quad a > 0$	$\frac{2a}{a^2 + (2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\begin{cases} 1 - \frac{\lvert t\rvert}{T}, & \lvert t\rvert < T \\ 0, & \lvert t\rvert \geq T \end{cases}$	$T\,\mathrm{sinc}^2(fT)$
$\delta(t)$	1
1	$\delta(f)$
$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$
$\exp(j2\pi f_c t)$	$\delta(f - f_c)$
$\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$
$\sin(2\pi f_c t)$	$\frac{1}{2j}[\delta(f - f_c) - \delta(f + f_c)]$
$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
$\frac{1}{\pi t}$	$-j\,\mathrm{sgn}(f)$
$u(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$\sum_{i=-\infty}^{\infty} \delta(t - iT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$

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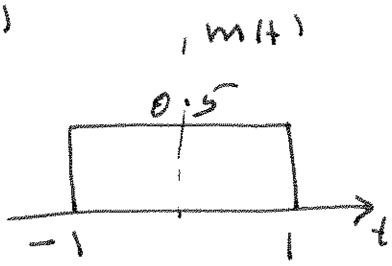
Solution to Final Exam

Problem 1 $\quad m(t) = \begin{cases} 0.5 & -1 \le t \le 1 \\ 0 & \text{o.w} \end{cases}$



a. $E = \int_{-1}^{1} (m(t))^2 \, dt = \int_{-1}^{1} (0.5)^2 \, dt$

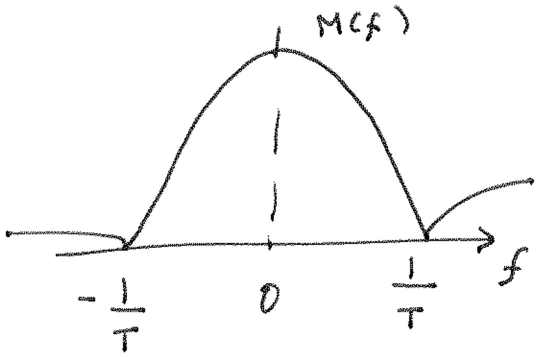
$= 0.5$

b. $m(t) = A \operatorname{rect}\left(\frac{t}{T}\right) \Rightarrow M(f) = A\,T \operatorname{sinc} f T$

$A = 0.5$

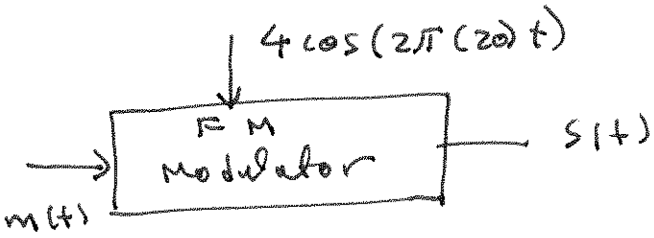
$T = 2$

$\Rightarrow M(f) = (0.5)(2) \operatorname{sinc}(2f)$



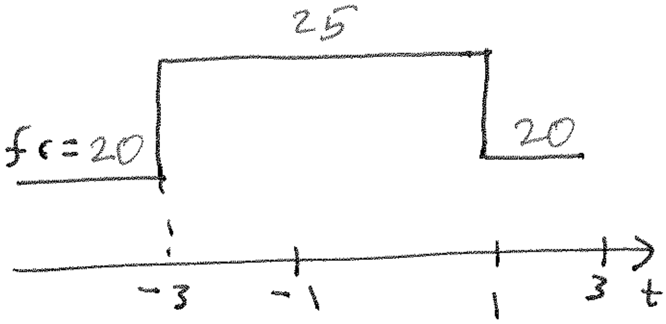
$B.W = \frac{1}{T} = \frac{1}{2}$

c.



$f_i(t) = f_c + k_f \, m(t)$

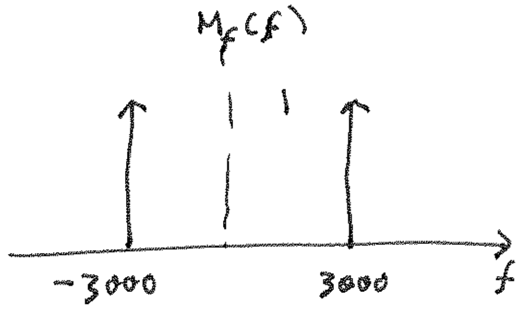
$= 20 + (10)\, m(t)$



Problem 2

$m(t) = 2\cos(2\pi(3000)t) + 4\cos(2\pi\, 16000\, t)$

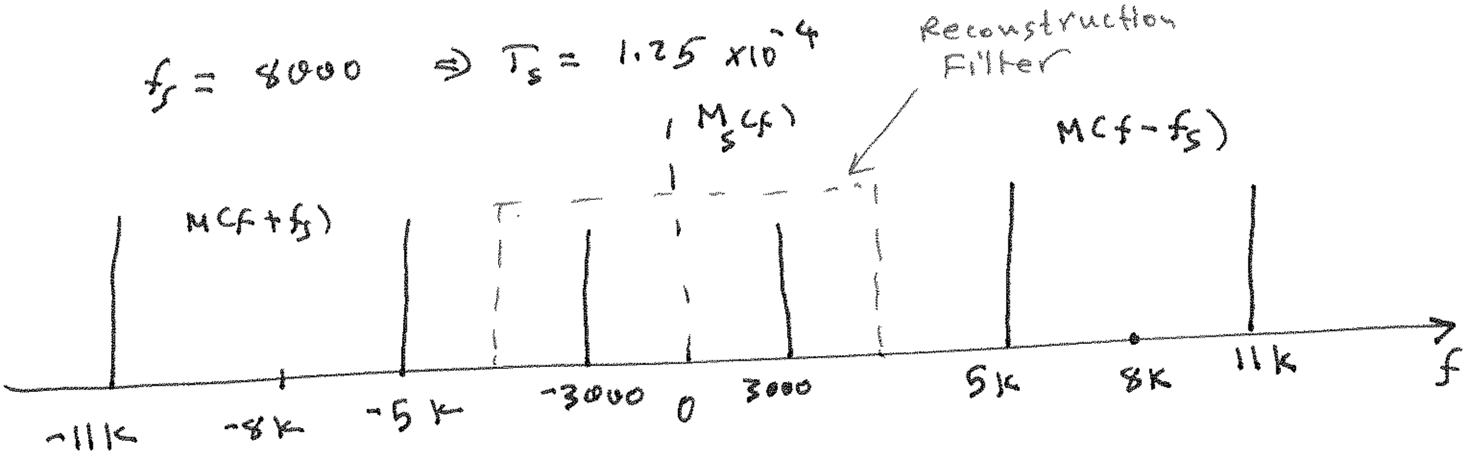
a. $m_f(t) = 2\cos(2\pi(3000)t)$



b. $M_f(f) = \delta(f-3000) + \delta(f+3000)$

c. $M_s(f) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} M_f(f - kf_s)$

$f_s = 8000 \Rightarrow T_s = 1.25 \times 10^{-4}$



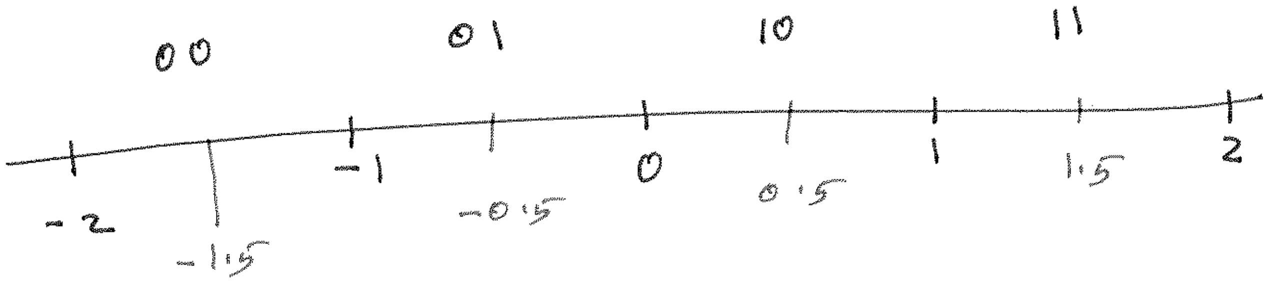
d. Reconstruction Filter: Low pass filter

e. $3000 < B.W < 5000$

f. $m_R(t) = f_s\, m_f(t)$

$m_R(t) = (8000)(2)\cos 2\pi(3000)t$

Problem 3



$f_s = 2 \times 4000 = 8000\,Hz \Rightarrow T_s = 0.125\,ms$

a. Time	0	0.125	0.25	0.375	0.5
sampled values (part a.)	1.25	-1.45	0.9	-0.45	-0.2
b. Quantised values (part b.)	1.5	-1.5	0.5	-0.5	-0.5
Binary representation (part c)	11	00	10	01	01

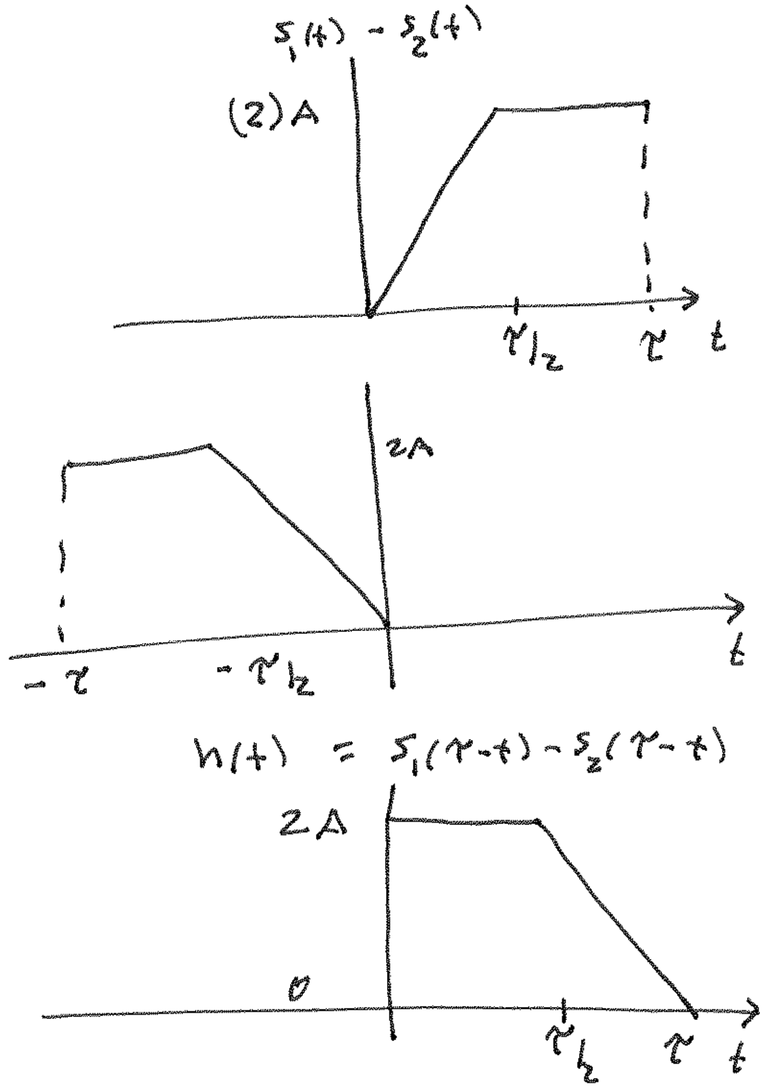
d. $R_b = (f_s)(\#\text{ of bits/ quantised sample})$

$= 2 \times 4000 \times 2 = 16{,}000$ bits/sec

e. $B.W = 2 \times R_b$

$= 2 \times 16{,}000 = 32{,}000\ Hz$

Problem 4



a. $h(t) = s_1(\tau - t) - s_2(\tau - t)$

b. $\lambda^* = \frac{1}{2}(E_1 - E_2)$

since $E_1 = E_2$

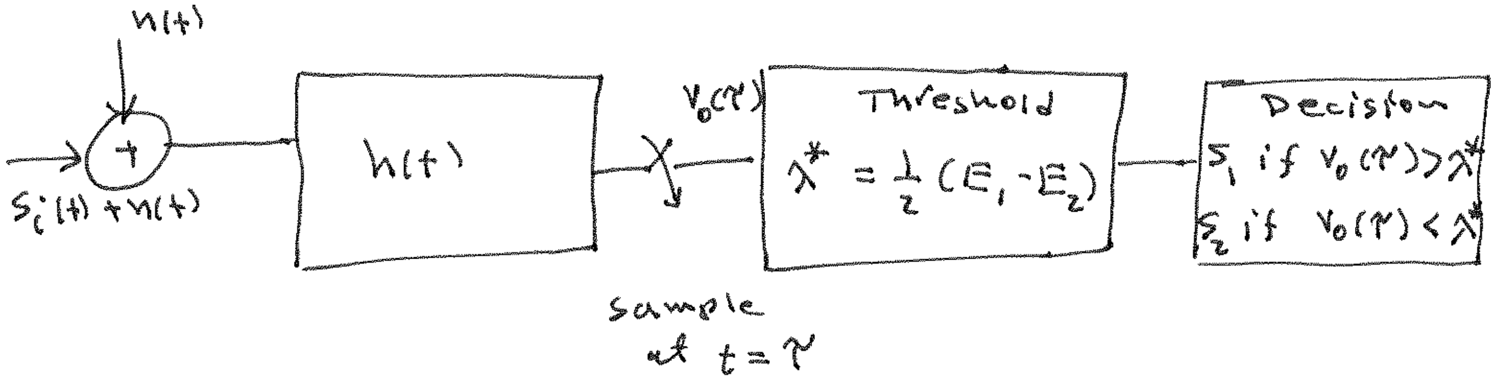
$\Rightarrow \lambda^* = 0$

Find E_1

$$E_1 = \int_0^{\tau/2} \left(A\,\frac{2t}{\tau}\right)^2 dt + \int_{\tau/2}^{\tau} A^2\, dt$$

$$= \frac{1}{6} A^2 \tau + A^2 \frac{\tau}{2}$$

$$E_1 = \frac{2}{3} A^2 \tau$$



Problem 5

a. $s_1(t) = 4\cos 2\pi f_1 t \qquad 0 \leqslant t \leq \tau$

$s_2(t) = 4\cos 2\pi f_2 t \qquad 0 \leqslant t \leq \tau$

a. $$P(E) = Q\left(\sqrt{\frac{\int_0^\tau (s_1(t) - s_2(t))^2}{2N_0}}\right)$$

$$\int_0^\tau (s_1(t) - s_2(t))^2\,dt = \int_0^\tau s_1(t)^2\,dt + \int_0^\tau s_2(t)^2\,dt - 2\int_0^\tau \cancelto{0}{s_1(t)s_2(t)}\,dt$$

$$= A^2\frac{\tau}{2} + A^2\frac{\tau}{2} = A^2\tau$$

$$P(E) = Q\left(\sqrt{\frac{A^2\tau}{2N_0}}\right) = Q\left(\sqrt{\frac{8\tau}{N_0}}\right)$$

b. $$P(E) = Q\left(\sqrt{\frac{8\tau}{0.001}}\right) \leqslant 8.8417 \times 10^{-5}$$

$$Q(x) \leqslant 8.8417 \times 10^{-5} \Rightarrow x = 3.75$$

$$3.75 = \sqrt{\frac{8\tau}{0.001}} \Rightarrow 14.0625 = \frac{8\tau}{0.001}$$

$$\tau = 1.7578 \times 10^{-3} \Rightarrow \boxed{R_b \leqslant 569 \text{ bits/sec}}$$

c.

$B.W = (f_1 - f_2) + 2R_b$

$= [(8-2) + 2]^k$

$\boxed{B.W = 8\ \text{KHZ}}$